



RSI Technical Note 040

Noise in Temperature Gradient Measurements

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1 Introduction

The purpose of this technical note is to provide a method for estimating the noise in measurements of the gradient of temperature fluctuations obtained with an RSI instrument. The method relies on functions in the ODAS Matlab Library and the user must be familiar with this collection of functions, which are documented in the ODAS Matlab Library User Guide. The functions of particular concern are;

- `odas_p2mat` which is used to convert raw data into physical units and to save them into a `mat`-file.
- `quick_look` which is used to visualize data and will call `odas_p2mat`, if necessary, to create a `mat`-file in physical units.

In addition, the RSI Technical Note 005, provides a detailed description of how data from shear probes, thermistors and micro-conductivity sensors are converted into physical units.

2 Noise sources

The thermistor signal, $N_{T,dT}$, is the result of four stages of analog and digital processing (Figure 1). Each stage contributes noise to the data. The stages are;

1. Thermistor bridge amplification.
2. Pre-emphasis – frequency-dependent amplification that compensates for the $f^{-5/3}$ spectral characteristic of turbulence signals.
3. Anti-aliasing stage — a cascade of two fourth-order low-pass filters to suppress aliasing.
4. Sampling stage — sampling by a 16-bit analog-to-digital converter (ADC).

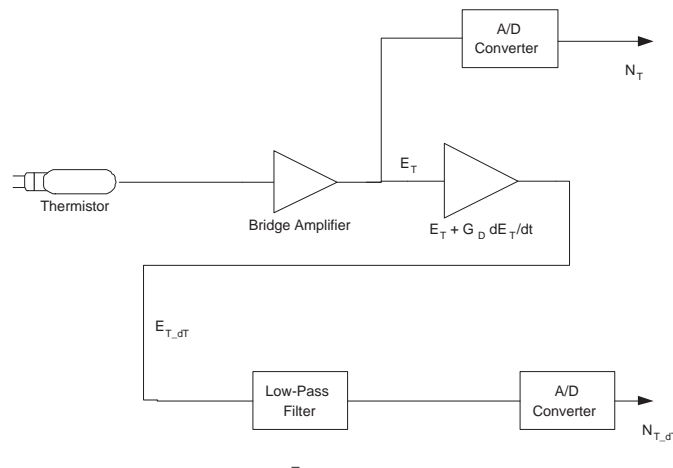


Figure 1: Thermistor amplifier schematic.

Amplification is achieved using operational amplifiers that have both “input voltage noise” and “input current noise”. However, for the range of resistance values used in the RSI thermistor circuit, the noise is entirely dominated by the input noise voltage. The output noise of any stage equals its input voltage noise boosted by the gain of that stage. The output voltage noise of one stage adds to the input voltage noise of the next stage.

2.1 Thermistor bridge amplification

The spectrum of the amplifier input voltage noise of the first stage of the thermistor circuit is

$$\phi_a(f) = 2a^2 \frac{f_a}{f} \left(1 + \left(\frac{f}{f_a} \right)^2 \right)^{1/2} \quad (1)$$

where the parameters are listed in Table 1. For very high frequencies ($f \gg f_a$), the noise is frequency-independent and equal to $2a^2$. For very low frequencies ($f \ll f_a$), the noise rises inversely with frequency. The turning point, f_a , that separates these two ranges is called the flicker-noise knee frequency.

The thermal (or Johnson) noise of the thermistor, including the three resistors in the Wheatstone bridge, is

$$\phi_R(f) = 4k_B T R_T \sim 4.8 \times 10^{-17} \text{ V}^2 \text{ Hz}^{-1} \quad (2)$$

where $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$ is Boltzmann’s constant, $R_T \sim 3000 \Omega$ is the bridge resistance, and the temperature T , is 290 K. The thermal noise is independent of frequency. The thermal noise is the lower limit of resolution imposed by thermodynamics on all resistance-type thermometers – it is the theoretical limit achieved by noiseless electronics.

The amplifier input voltage noise and the thermal noise add and are multiplied by the frequency-independent gain of the first stage, G_1 . Thus, the output voltage noise of the thermistor bridge amplifier is

$$\phi_1(f) = G_1^2 [\phi_a + \phi_R] \quad (3)$$

2.2 Pre-emphasis

The second stage of the electronics provides the frequency-dependent gain of pre-emphasis. The operational amplifier used for pre-emphasis is different from the one used for bridge amplification. The amplifier input voltage noise of this stage (excluding any contribution from the first stage) is

$$\phi_b(f) = 2b^2 \frac{f_b}{f} \left(1 + \left(\frac{f}{f_b} \right)^2 \right)^{1/2} \quad (4)$$

where both the high-frequency noise, $2b^2$ and the flicker-noise frequency knee, f_b , are larger than for the previous stage (see Table 1).

The output noise of the first stage adds to this noise, and their sum is multiplied by the gain of the pre-emphasis stage. The output voltage noise of the pre-emphasis stage is thus

$$\begin{aligned}\phi_2(f) &= G_2^2 [\phi_1 + \phi_b] \\ &= G_2^2 [G_1^2 \phi_a + G_1^2 \phi_R + \phi_b]\end{aligned}\tag{5}$$

where

$$G_2^2(f) = 1 + (2\pi G_D f)^2\tag{6}$$

is the frequency-dependent gain of the pre-emphasis stage, and $G_D \approx 0.94$ s is the gain of the differentiator used to provide this pre-emphasis.

2.3 Anti-aliasing Stage

After pre-emphasis, the thermistor signal is low-pass filtered for anti-aliasing purposes. The frequency dependent gain-squared of this stage is

$$G_{AA}^2(f) = \left[1 + \left(\frac{f}{f_{AA}} \right)^8 \right]^{-2}.\tag{7}$$

The effective ‘half-power’ response frequency is typically 100 Hz, and there is little attenuation for lower frequencies. Because the anti-aliasing filter provides no gain, its noise is comparatively low and can be neglected. However, the filter modifies the frequency dependence of the circuit noise, $\phi_2(f)$, by limiting its bandwidth. Therefore, the noise at the output of the anti-aliasing filter is

$$\begin{aligned}\phi_{AA}(f) &= G_{AA}^2(f) \phi_2(f) \\ &= G_{AA}^2(f) G_2^2(f) [G_1^2 \phi_a + G_1^2 \phi_R + \phi_b]\end{aligned}\tag{8}$$

2.4 Sampling Stage

Because the sampled data have to be quantized using a finite number of bits, no sample is ever exactly equal to the continuous-domain signal. An ideal sampler has an error that is within $\pm 1/2$ step size of the sampler. However, real samplers never attain the ideal limit. An ideal sampler has a variance of

$$\frac{\delta_s^2}{12}\tag{9}$$

where δ_s is the step size of the sampler¹. The sampling noise is spread uniformly over the Nyquist band and, so the sampling noise spectrum is

$$\hat{\phi}_s = \frac{\delta_s^2}{12} \frac{2}{f_s}\tag{10}$$

¹The sampler produces whole numbers. δ_s is the value in physical units (in this case volts) representing a change of one unit of the sampler.

Item	Value	Description
a	$4.0 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}}$	First-stage amplifier input noise at high frequency.
f_a	19 Hz	First-stage flicker noise knee frequency.
b	$8.0 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}}$	Second-stage amplifier input noise at high frequency.
f_b	42 Hz	Second-stage flicker noise knee frequency.
k_B	$1.38 \times 10^{-23} \text{ J K}^{-1}$	Boltzmann's constant.
R_0	$3000 \Omega \pm 0.1 \%$	Bridge resistors.
R_T	3000Ω	Typical thermistor resistance.
T	290 K	Typical absolute temperature.
G_1	$6 \pm 0.1 \%$	Voltage gain of first stage.
G_D	0.94 s	Pre-emphasis gain factor. Calibrated by RSI to 0.2%.
f_{AA}	110 Hz $\pm 2 \%$	Anti-aliasing filter frequency parameter.
δ_s	$62.5 \times 10^{-6} \text{ V} \pm 0.1 \%$	Typical voltage step size of RSI sampler.
γ	3	Non-ideal sampler noise factor.

Table 1: Noise parameters of the RSI thermistor circuit. Unless otherwise specified, the values are nominal and typical, but manufacturing variability can be as large as a factor of 2.

where f_s is the sampling rate (usually $f_s = 512 \text{ Hz}$). All real samplers have noise that is larger than (10). The noise of the sampler developed by RSI is

$$\phi_s = \gamma \frac{\delta_s^2}{12 f_s} . \quad (11)$$

where $\gamma \approx 3$.

The sampling noise adds to the output of the sampler. Thus, the sampled data have a noise spectrum of

$$\phi_V(f) = G_{AA}^2(f) G_2^2(f) [G_1^2 \phi_a + G_1^2 \phi_R + \phi_b] + \gamma \frac{\delta_s^2}{12 f_s} \quad (12)$$

when the samples are expressed in units of volts. To derive the noise in terms of the raw data values (the whole numbers produced by the sampler), we must scale the spectrum of (12) by the square of the step size of the sampler. The RSI samplers are usually designed to span a range of 4.096 V using 16-bit samples. Thus, the step size of the RSI sampler is

$$\delta_s = \frac{4.096 \text{ V}}{2^{16}} = 62.5 \times 10^{-6} \text{ V} . \quad (13)$$

The noise spectrum, in terms of dimensionless whole numbers (the samples), is thus

$$\begin{aligned} \phi_N(f) &= \delta_s^{-2} \phi_V(f) \\ &= \delta_s^{-2} G_{AA}^2(f) G_2^2(f) [G_1^2 \phi_a + G_1^2 \phi_R + \phi_b] + \frac{\gamma}{6 f_s} . \end{aligned} \quad (14)$$

This model of the noise fits well to the routine calibration tests conducted at RSI (Figure 2). For the noise tests, the thermistor is replaced by a fixed resistor of 3000 Ω .

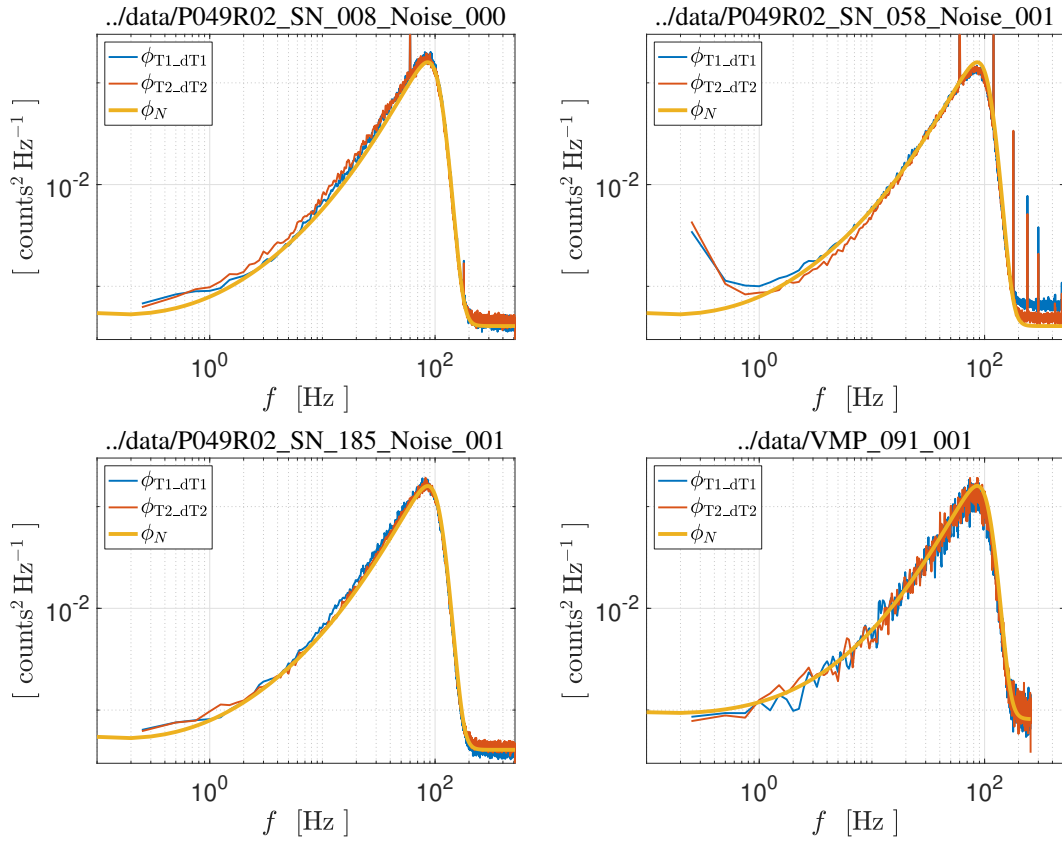


Figure 2: The spectra of measured noise from both thermistor channels (blue and red) and the analytic model of the noise (14) (gold), for four different instruments that were sampled at a variety of rates.

3 Temperature gradient noise

The data samples that you have collected through the thermistor channels, N_{T_dT} , must be further processed to produce a signal that is the spatial gradient of temperature in physical units of $^{\circ}\text{C m}^{-1}$, or equivalently K m^{-1} . The conversion is a three step process.

1. First, the data are high-pass filtered and scaled to make the signal a time-derivative of (raw) temperature, dN_T/dt .
2. Second, this time derivative is converted into a spatial gradient of raw temperature by dividing it by the speed of profiling, i.e. $\partial N_T/\partial z = W^{-1}dN_T/dt$, where W is the speed of profiling.
3. Third, this spatial gradient is converted into physical units using equation (28) in the TN-005.

The spectrum of raw noise (14) must be similarly transformed.

The magnitude-squared of the scaled high-pass filter is

$$G_{HP}^2(f) = \frac{1}{G_D^2} \left[\frac{(2\pi G_D f)^2}{1 + (2\pi G_D f)^2} \right]. \quad (15)$$

The redundant use of G_D is to make it clear that the term in square braces is a standard first-order high-pass filter with a cut-off frequency of $f_c = 1/(2\pi G_D)$. The factor of G_D^{-2} is the scale factor. The reason that this filter converts the raw noise spectrum of pre-emphasized temperature into a spectrum of the time derivative of temperature is that

$$G_{HP}^2(f) G_2^2(f) = (2\pi f)^2 \quad (16)$$

which equals the magnitude-squared of the Fourier transform of a time differentiator. The manipulation of the spectrum is the same as the manipulation of the time series.

Thus, the spectrum of the rate of change of the raw pre-emphasized temperature signal is

$$\begin{aligned} \phi_{dN/dt}(f) &= G_{HP}^2 \phi_N(f) \\ &= \delta_s^{-2} (2\pi f)^2 G_{AA}^2(f) [G_1^2 \phi_a + G_1^2 \phi_R + \phi_b] + \frac{\gamma}{6f_s} G_{HP}^2(f) \\ &\approx \delta_s^{-2} (2\pi f)^2 [G_1^2 \phi_a + G_1^2 \phi_R + \phi_b] + \frac{\gamma}{6f_s G_D^2} \end{aligned} \quad (17)$$

where the approximation applies to the frequency range of $(2\pi G_D)^{-1} \ll f \ll f_{AA}$.

Dividing the frequency spectrum of the *time* derivative of the noise (17) by W^2 produces the frequency spectrum of the *spatial* gradient of raw temperature noise, namely,

$$\begin{aligned} \phi_{\nabla N}(f) &= W^{-2} \phi_{dN/dt}(f) \\ &= W^{-2} \left[\delta_s^{-2} (2\pi f)^2 G_{AA}^2(f) [G_1^2 \phi_a + G_1^2 \phi_R + \phi_b] + \frac{\gamma}{6f_s} G_{HP}^2(f) \right] \end{aligned} \quad (18)$$

The scale factor that converts (18) into physical units can be gleaned from equation (28) in TN-005. It equals 3.0×10^{-6} , but is slightly dependent on temperature itself and on the temperature-coefficients of resistance of the thermistor in question. It varies by 20% over the oceanic temperature range. Although, one can use (18) and the scale factor to compute the spectrum of temperature-gradient noise, this is tedious and, consequently, a function has been developed for deriving the temperature gradient noise spectrum.

4 Matlab algorithm

RSI has developed the Matlab function `gradT_noise_odas` to compute the frequency spectrum of the noise in the measurement of the gradient of temperature fluctuations. This function uses the noise model of (18) and the very same method that converts a raw temperature signal into the gradient of temperature in physical units. It is not an approximation, such as using a single scale factor. It applies to the entire oceanic temperature range and to all thermistors.

The syntax for calling this function is

```
gradT_noise = ...
    gradT_noise_odas (T_dT, T_dT_name, speed, F, setupfilestr, varargin);
```

The input arguments are as follows.

- `T_dT` – a vector of the raw pre-emphasized data segment that was used to estimate the frequency spectrum of the gradient of temperature in physical units. This vector usually has the name `T1_dT1` or `T2_dT2`, and the corresponding temperature gradient in physical units usually has the name `gradT1` or `gradT2`.
- `T_dT_name` – a string containing the name of `T_dT`. It is required to find the calibration coefficients for the thermistor in question. This string is usually `'T1_dT1'` or `'T2_dT2'`, but the function will also recognize the abbreviated names of `'T1'` and `'T2'`.
- `speed` – a scalar or vector that contains the speed of profiling for the data segment `T_dT`. If it is a vector, its average value is used to calculate the noise spectrum.
- `F` – a vector of frequency at which to calculate the noise spectrum. Usually, this vector is returned by the function that was used to calculate the spectrum of the gradient of temperature in physical units. Using such a frequency vector provides the noise spectrum at exactly the same frequency values as those of the spectrum of the gradient of temperature. However, any vector of frequency values can be used.
- `setupfilestr` – a string containing the configuration file that was used to acquire the data. This string is a standard product of the conversion of a raw data file into a mat-file containing your data in physical units. The name of this string is literally `setupfilestr`.
- `varargin` – either a structure or a collection of string-value pairs that can be used to override the default values of the thermistor noise model given in Table 1.

The output of the function is the noise frequency spectrum `gradT_noise` in physical units ($^{\circ}\text{C}^2 \text{m}^{-2} \text{Hz}^{-1}$).

The default values of the noise model can be obtained by calling the function without any input arguments. For example, using the syntax

```
noise_info = gradT_noise_odas
noise_info =
```

```
struct with fields:
```

```

    E_n: 4.0000e-09
    E_n2: 8.0000e-09
    K_B: 1.3820e-23
    R_0: 3000
    T_K: 295
    f_AA: 110
    fc: 18.7000
    fc_2: 42
    gamma_RSI: 3
    make_figure: 0
```

returns the default model parameters. All other parameters can be deduced from the `setupfilestr` string. If you want to change any of the parameters, then you can edit the values of the fields, and use the modified `noise_info` as an input argument to the function. For example,

```
noise_info = gradT_noise_odas;
noise_info.f_AA = 220;
gradT_noise = ...
    gradT_noise_odas (T_dT, T_dT_name, speed, F, setupfilestr, noise_info);
```

will change the anti-aliasing frequency to 220 Hz.

The field `make_figure` is false by default. If you would like to see a plot of the noise spectrum, then you can set this field to true. For example

```
gradT_noise = ...
    gradT_noise_odas (T_dT, T_dT_name, speed, F, setupfilestr, 'make_figure', true);
```

will produce a graph similar to Figure 3.

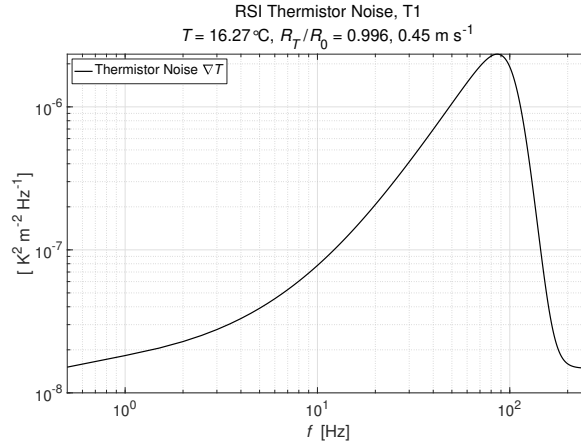


Figure 3: The frequency spectrum of the temperature gradient noise in physical units, using data collected from a glider in Puget Sound.

4.1 Temperature gradient noise example

I will use a segment of data, collected with a Sea-glider in Puget Sound, WA., to demonstrate a usage of the function `gradT_noise_odas` (Figure 4). The time series of the raw pre-emphasized data (upper panel) is a linear combination of the temperature signal and its time rate of change. The along-path gradient of temperature (lower panel) was obtained using standard processing with the ODAS Matlab Library (see TN-039 for details).

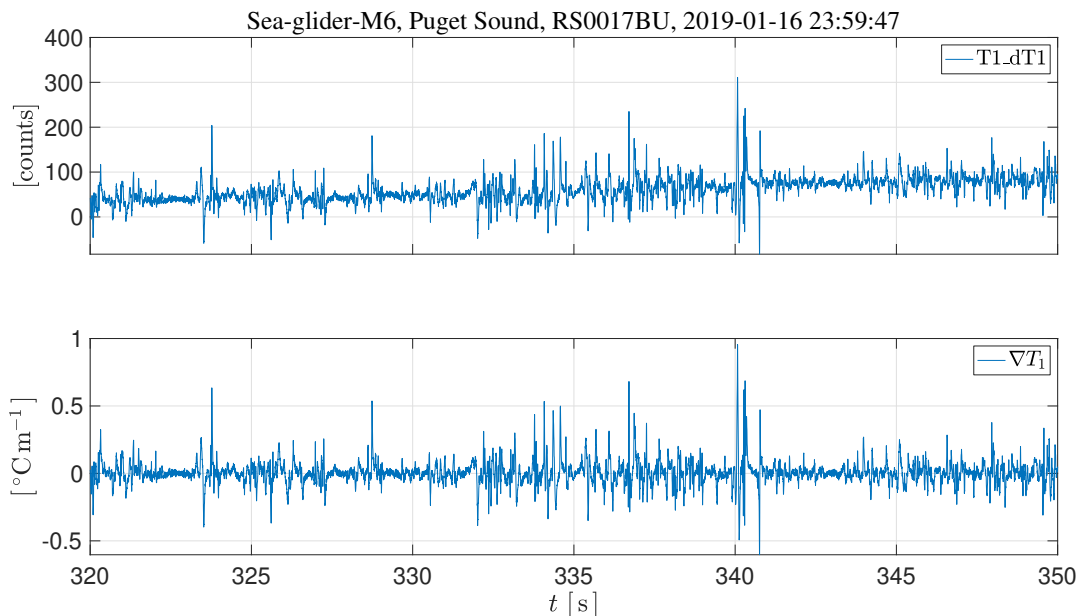


Figure 4: A segment of the time series of raw pre-emphasized data collected from a Sea-glider in Puget Sound (upper panel), and the same data after conversion into physical units (lower panel). The glider speed was 0.45 m s^{-1} .

The frequency spectrum of the gradient of temperature was obtained with standard processing (Figure 5, blue). The noise spectrum in physical units was obtained using the function `gradT_noise_odas` with default parameters (Figure 5, red). The measured oceanic spectrum is well above the noise spectrum for all frequencies smaller than ~ 60 Hz. The peak in the measured spectrum is slightly lower than the noise spectrum (near ~ 90 Hz), and the high-frequency sampling noise ($f > 200$ Hz) is also slightly lower than noise spectrum using default values. The default values are based on nominal and typical values, but manufacturing variability will lead to slight differences in the actual values of the parameters. Often, the actual noise of a component is less than its specification. Unusually high noise values are corrected by various quality-control protocols during the manufacturing of an instrument.

I have adjusted the default values used by the `gradT_noise_odas` function. At the very highest frequencies, the dominant source of noise is the sampling noise. I decreased the noise factor γ from 3 to 2 to bring the sampling noise model into closer agreement with the observations for $f > 200$ Hz (Figure 5, gold). It is not unusual for γ to be as small as 2.

For the frequency range of 10 to $f_{AA} = 110$ Hz, the dominant source of noise is the high-frequency voltage noise, E_n , of the operational amplifier in the bridge circuit (see Table 1). Therefore, I decreased E_n by a factor of two to $2 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}}$ to lower the peak of the noise spectrum. A reduction of E_n by a factor of 2 is uncommon, but does occur occasionally.

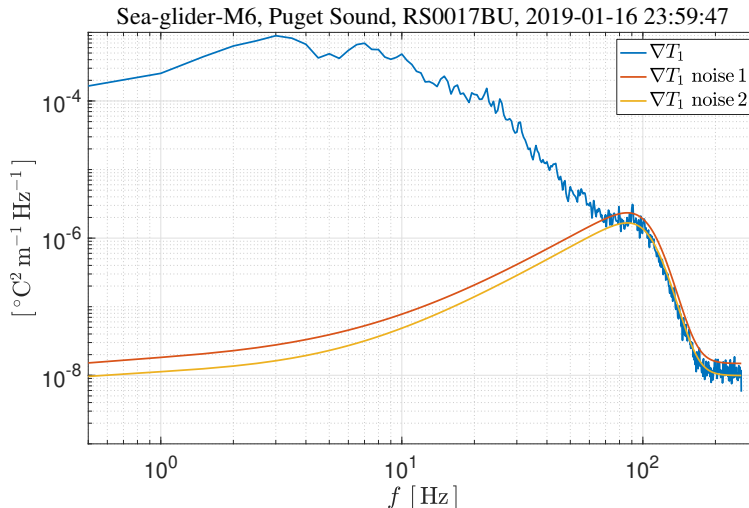


Figure 5: Blue – The spectrum of the gradient of temperature obtained with thermistor number 1 using the time-series shown in Figure 4. Red – the noise spectrum using default parameters. Gold – the noise spectrum using slightly modified parameters described in the text.

If your measured spectrum is dominated by noise over some range of frequency (which is always in the high-frequency range of the spectrum), then you may be able to adjust the model to make it agree more closely with your observations. In addition, you can always collect some data using the ‘test’ probes that should be installed in your instrument for shipping, storage and for laboratory testing. You can then use the function `gradT_noise_odas` to evaluate the noise of the thermistor channels for a particular speed of interest. Be sure that

the test probes are labeled T1, or T2, or T because there is a 3000 Ω resistor embedded within these test probes to bring the outputs of thermistor channels to their mid-range.

If you think that your instrument is excessively noisy, you can use the function `bench_test` to produce a noise spectrum of the raw pre-emphasized signals and compare these to the noise spectrum produced at RSI before your instrument was shipped.

End of document
