RSI Technical Note 042

Noise in Shear Probe Measurements

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1 Introduction

In this technical note, we develop a model of the electronic noise of the shear-probe circuit. The model is used to test the performance of an instrument by collecting data with 'test' probes, in place of real probes, because shear probes are extremely sensitive to vibrations and 50/60 Hz contamination from power mains. Such a measurement is usually called a bench test. The model is also used to estimate the electronic noise in oceanic data collected with real shear probes. The test probe makes no connection to the electronics (it is an open circuit with negligible capacitance, $C_p = 0\,\mathrm{nF}$). Real probes have a capacitance ($C_p \approx 1\,\mathrm{nF}$) and this increases slightly the noise produced by the electronic circuitry. RSI Technical Note 005 describes how shear-probe data are converted into physical units.

The noise sources for the shear channel are identified in Section 2 where we develop a model of the noise. This model is compared against bench-test data in Section 3. Brief instructions on testing the shear channel noise of an instrument are provided in Section 4, and the conversion of the noise model into physical units of shear (for comparison against oceanic measurements of shear) is given in Section 5. A Matlab function to compute the noise model is described in Section 6.

2 Noise Sources

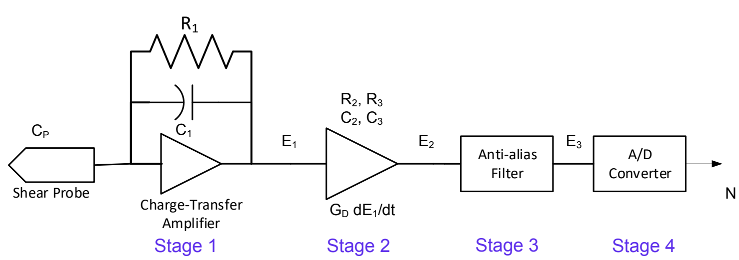

Figure 1: A block diagram of the shear probe circuit in a typical instrument. The resistors and capacitors, R_2, R_3, C_2 and C_3, control the differentiator in stage 2.

The shear probe signal is the result of four stages of analog and digital processing (Figure 1). All stages contribute noise to the data and the contributions from three of the four stages are frequency dependent. The stages are:

1. Charge Removal – uses a capacitor to remove the charge produced by the shear probe to generate a voltage, E_1. The shear-probe capacitance is $C_p \approx 1\,\mathrm{nF}$.
2. Pre-emphasis – frequency-dependent amplification that compensates for the $f^{-5/3}$ spectral characteristic of turbulence signals. It produces the signal $E_2 = G_D \,\mathrm{d}E_1/\mathrm{d}t$.
3. Anti-aliasing – a cascade of two fourth-order low-pass filters to suppress aliasing (E_3).
4. Sampling – sampling by a 16-bit analog-to-digital converter (ADC), which produces a numeric signal N.

The output noise of any stage equals its input voltage noise boosted by the gain of that stage. The output voltage noise of one stage adds to the input voltage noise of the next stage.

2.1 Stage One – Charge Removal

The removal of the charge from the shear probe is accomplished using a charge-transfer amplifier (LTC-6240). The total noise is the sum of the input voltage noise (ϕ_{V_1}), the input current noise (ϕ_{I_1}), and the thermal noise generated by the resistor (ϕ_{R_1}). The noise gain of the first stage, $G_1(f)$, is unity when the shear probe is disconnected, which is the configuration for a typical noise test.

The input voltage noise was estimated from the specification sheet of the LTC-6240 operational amplifier and is given by

$$\phi_{V_1}(f) = E_1^2 \frac{f_c}{f} \left[1 + \left(\frac{f}{f_c} \right)^2 \right]^{1/2} , \tag{1}$$

where both the high frequency noise, E_1^2, and the flicker-noise knee frequency, f_c, are given in Table 1. The model is plotted in Figure 2. For very low frequencies ($f \ll f_c$), the noise has an f^{-1} frequency dependence and is often called the "flicker" noise. For very high frequencies ($f \gg f_c$), the noise is frequency-independent and equal to E_1^2. The turning point from flicker to frequency-independent noise, f_c, is usually called the flicker frequency "knee".

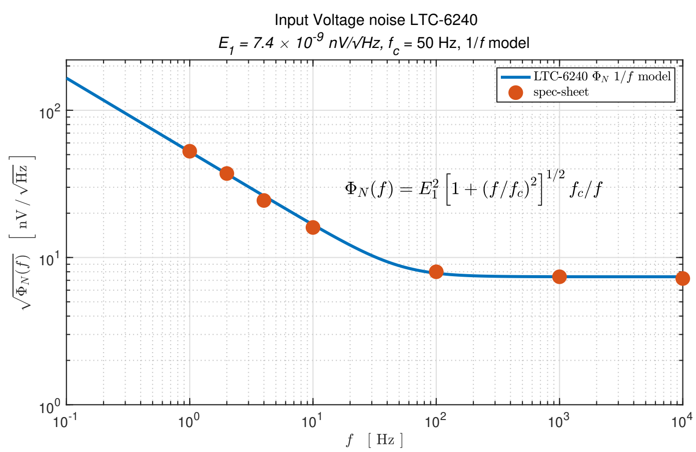

Figure 2: Mathematical model (blue) used to represent voltage noise as a function of frequency for the LTC-6240. The red markers are values gleamed from the specification sheet.

The input current noise equals the current noise times the impedance of the RC feedback, which is given by

$$\phi_{I_1}(f) = I_1^2 \frac{R_1^2}{1 + (\omega R_1 C_1)^2} , \tag{2}$$

Item	Value	Description
k_B	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$	Boltzmann's constant
T	$295\,\mathrm{K}$	Absolute temperature
E_1	$7.4 \times 10^{-9}\,\mathrm{V}/\sqrt{\mathrm{Hz}}$	LTC-6240 input voltage noise at high frequency
f_c	$42\,\mathrm{Hz}$	LTC-6240 flicker noise knee frequency
I_1	$0.56 \times 10^{-15}\,\mathrm{A}/\sqrt{\mathrm{Hz}}$	LTC-6240 input current noise
R_1	$1 \times 10^{9}\,\Omega$	First stage resistor
R_2	$499\,\Omega$	Second stage resistor
R_3	$1 \times 10^{6}\,\Omega$	Second stage resistor
C_1	$1.5 \times 10^{-9}\,\mathrm{F}$	First stage capacitor
C_2	$0.94 \times 10^{-6}\,\mathrm{F}$	Second stage capacitor
C_3	$470 \times 10^{-12}\,\mathrm{F}$	Second stage capacitor
C_p	$1 \times 10^{-9}\,\mathrm{F}$	Shear probe capacitance
f_{AA}	$110\,\mathrm{Hz}$	Anti-aliasing filter frequency
γ	2.5	Noise factor of the RSI sampler
f_s	512	Sampling rate

Table 1: Typical values of the noise parameters of an RSI shear-probe circuit.

where $\omega = 2\pi f$ is the angular frequency, I_1^2 is the frequency-independent input current noise of the operational amplifier and R_1 and C_1 are the resistance and capacitance, respectively, of the RC circuit. Typical values for the parameters are listed in Table 1.

The thermal (or Johnson) noise of the resistor, R_1, is filtered by the capacitor, C_1, to yield

$$\phi_{R_1}(f) = \frac{4k_BT}{R_1}\frac{R_1^2}{1+(\omega R_1 C_1)^2} \tag{3}$$

where k_B is Boltzmann's constant and T is the temperature in Kelvin.

The noise gain (amplification) of the first stage is

$$\begin{aligned} G_1^2(f) &= \left|1 + j\omega C_p Z_{R_1C_1}\right|^2 \\ &= \frac{1+(\omega R_1[C_1+C_p])^2}{1+(\omega R_1 C_1)^2} \\ &\approx \left(1+\frac{C_p}{C_1}\right)^2 = \frac{25}{9} \quad , \quad \text{for } \omega \gg (R_1C_p)^{-1} = 1\,\mathrm{rad\,s^{-1}} \end{aligned} \tag{4}$$

where $Z_{R_1C_1}$ is the parallel impedance of R_1 and C_1 in the feedback loop of the first-stage amplifier (Figure 1), and $C_p \approx 1\,\mathrm{nF}$ is the capacitance of the shear probe (Table 1). For most noise tests, $G_1^2(f) = 1$, because the shear probes are replaced by test probes with $C_p = 0\,\mathrm{nF}$.

The total output noise of the first stage is the sum of these three contributions (Figures 3 and 3, magenta), i.e.,

$$\phi_1(f) = G_1^2(f)\left[\phi_{V_1} + \phi_{I_1}\right] + \phi_{R_1} \ . \tag{5}$$

The thermal noise is independent of G_1. For all $f > 1\,\text{Hz}$, the input current noise, ϕ_{I_1}, is negligible. The thermal noise, ϕ_{R_1}, is the dominant source of noise for $f \lesssim 30\,\text{Hz}$, whereas for $f \gtrsim 30\,\text{Hz}$, ϕ_{V_1} is the dominant noise source (Figure 3). The main consequence of attaching a probe is a slight amplification of ϕ_{V_1} (Figure 4, blue) which raises the output noise slightly for $f \gtrsim 30\,\text{Hz}$ (magenta).

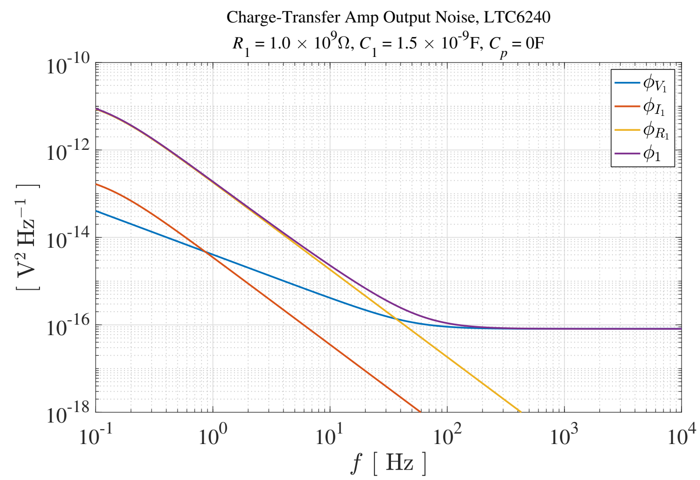

Figure 3: Noise contributions to the first stage output of the shear-probe circuit, when the probe is disconnected and $C_p = 0\,\text{nF}$.

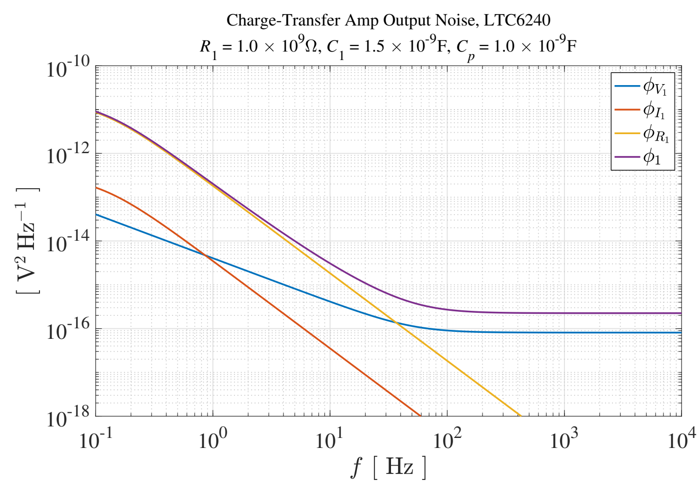

Figure 4: Same as Figure 3 but with a shear-probe of capacitance $C_p = 1\,\text{nF}$ connected to the circuit.

2.2 Stage Two – Pre-emphasis

The second stage of the electronics provides the frequency-dependent gain of pre-emphasis. This stage uses the same operational amplifier (LTC-6240) as the first stage. Thus, the input voltage noise is given by $\phi_{V2} = \phi_{V1}$ as defined in (1). The input current noise and the Johnson thermal noise are negligible because the resistors that are used in this stage, R_2 and

R_3, are 1000 times smaller than those in stage one (Table 1). The voltage noise from this stage gets added to the output noise of stage one, and their sum is amplified by the voltage gain, G_2^2, of stage 2 to yield

$$\begin{aligned} \phi_2(f) &= G_2^2 \left(\phi_1 + \phi_{V2}\right) \\ &= G_2^2 \left([G_1^2 + 1]\phi_{V_1} + G_1^2 \phi_{I_1} + \phi_{R_1} \right) \end{aligned} \tag{6}$$

where

$$G_2^2(f) = 1 + \frac{\left(\omega R_3 C_2\right)^2}{\left[1 + (\omega R_2 C_2)^2\right]\left[1 + (\omega R_3 C_3)^2\right]} \ . \tag{7}$$

The resistance (R_2 and R_3) and capacitance (C_2 and C_3) values are listed in Table 1.

2.3 Stage Three – Anti-aliasing

In the third stage, the shear-probe signal is low-pass filtered for anti-aliasing purposes. Because the anti-aliasing filter provides no gain, its noise is comparatively low and can be neglected. However, the filter modifies the frequency dependence of the circuit noise, by limiting its bandwidth. The gain-squared of this stage is given by

$$G_{AA}^2(f) = \left[1 + \left(\frac{f}{f_{AA}} \right)^8 \right]^{-2} \quad , \tag{8}$$

where f_{AA} is the effective 'half-power' response frequency (Table 1). Therefore, the output noise of the anti-aliasing filter is

$$\begin{aligned} \phi_3(f) &= G_{AA}^2(f)\phi_2(f) \\ &= G_{AA}^2 G_2^2 \left([G_1^2 + 1]\phi_{V_1} + G_1^2 \phi_{I_1} + \phi_{R_1} \right) \ . \end{aligned} \tag{9}$$

2.4 Stage Four – Sampling

Because the sampled data have to be quantized using a finite number of bits, no sample is ever exactly equal to the continuous-domain signal. An ideal sampler has an error that is within $\pm 1/2$ step size of the sampler. However, real a sampler[1] never attains the ideal limit. An ideal sampler has a variance of

$$\frac{\delta_s^2}{12}, \tag{10}$$

where δ_s is the step size of the sampler which is given by

$$\delta_s = V_{FS}/2^B \tag{11}$$

[1] Also called an analog-to-digital converter, or ADC.

where B is the number of bits of the sampler and V_{FS} is the full-scale voltage range of the sampler. Usually, $V_{FS} = 4.096\,\mathrm{V}$ and $B = 16$. The sampling noise is spread uniformly over the Nyquist band. So, the ideal sampling noise spectrum is

$$\hat{\phi}_S = \frac{\delta_s^2}{12}\frac{2}{f_s} \ , \tag{12}$$

where f_s is the sampling rate. All real samplers have noise that is larger than equation (12). The noise of the sampler developed by RSI is

$$\phi_S = \gamma\frac{\delta_s^2}{12}\frac{2}{f_s} \quad , \tag{13}$$

where $\gamma \approx 2.5$.

The total noise, ϕ_N in the shear-probe signals, in units of $\mathrm{V}^2\,\mathrm{Hz}^{-1}$, is

$$\begin{aligned} \phi_N(f) &= \phi_3 + \phi_S \\ &= G_{AA}^2 G_2^2 \left([G_1^2 + 1]\phi_{V_1} + G_1^2\phi_{I_1} + \phi_{R_1} \right) + \gamma\frac{\delta_s^2}{12}\frac{2}{f_s} \end{aligned} \tag{14}$$

In terms of samples (i.e. raw counts), the noise spectrum is

$$\begin{aligned} \Psi_N(f) &= \frac{1}{\delta_s^2}\phi_N(f) \\ &= \frac{1}{\delta_s^2} G_{AA}^2 G_2^2 \left([G_1^2 + 1]\phi_{V_1} + G_1^2\phi_{I_1} + \phi_{R_1} \right) + \frac{\gamma}{12}\frac{2}{f_s} \ . \end{aligned} \tag{15}$$

The voltage and noise spectra are plotted in Figures 5 and 6 for the parameters listed in Table 1. The noise at the output at each of the four stages are shown in Figures 7 and 8.

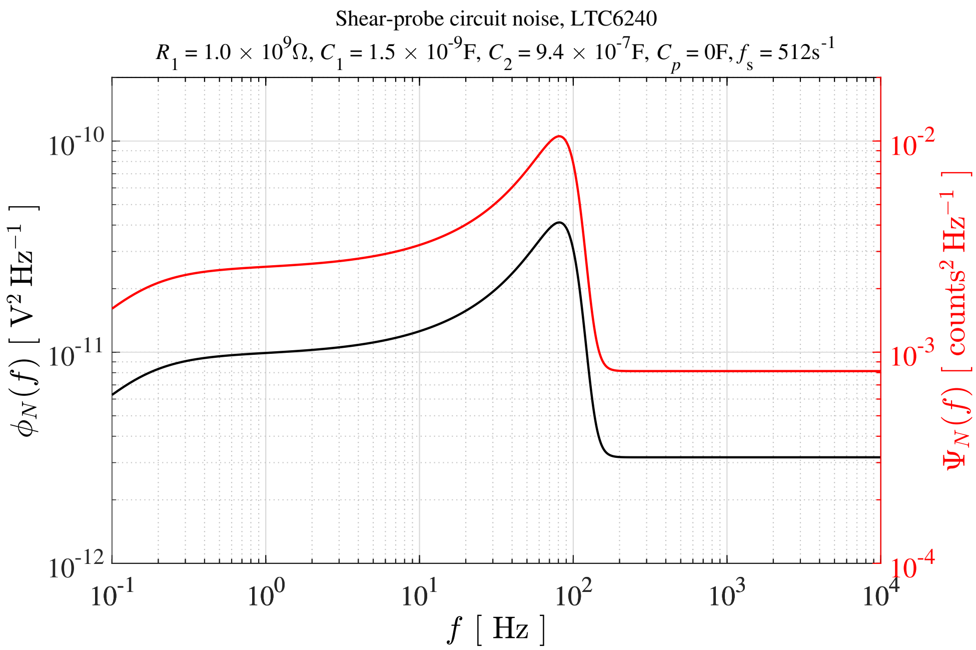

Figure 5: Model noise spectra, ϕ_N and Ψ_N, for the shear probe circuits in units of $\text{V}^2\,\text{Hz}^{-1}$ (black) and counts^2/Hz (red), respectively. The probe is disconnected.

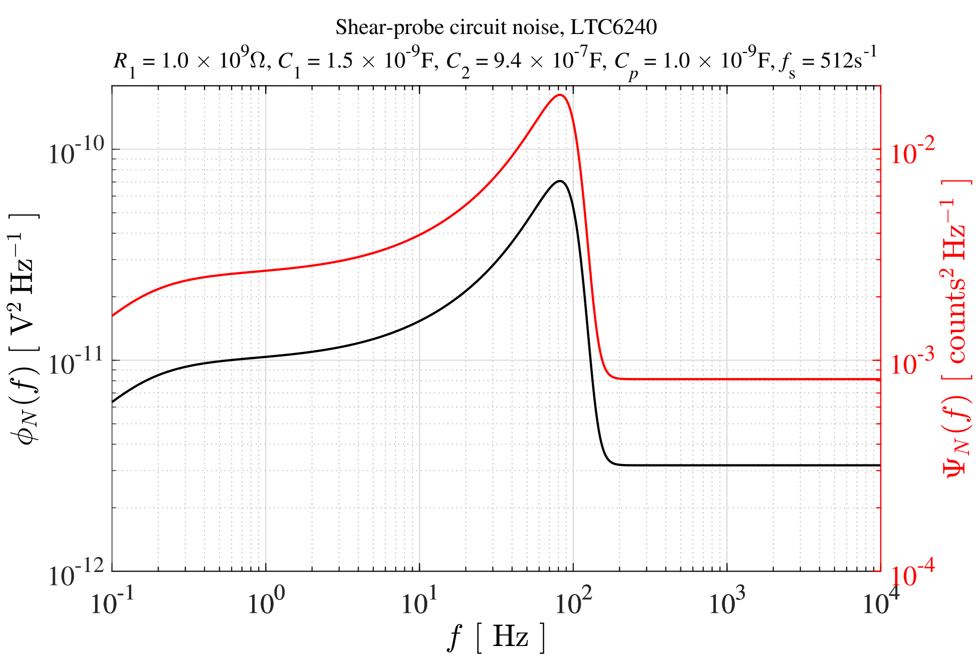

Figure 6: Same as Figure 5 but with shear-probe connected.

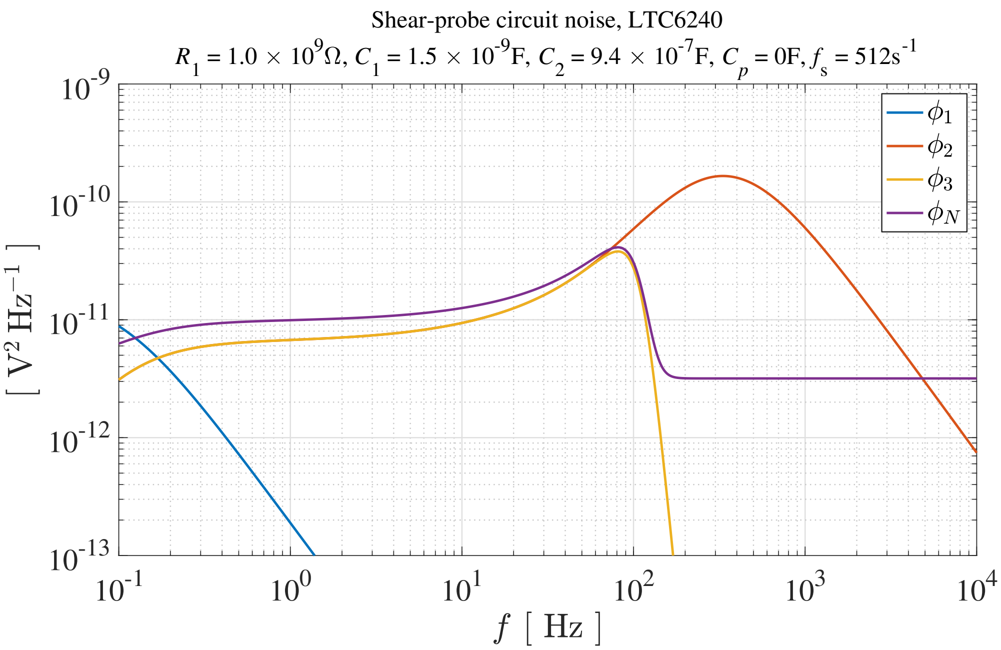

Figure 7: Model noise spectra at the output of each of the four stages as labelled in the legend. The probe is disconnected.

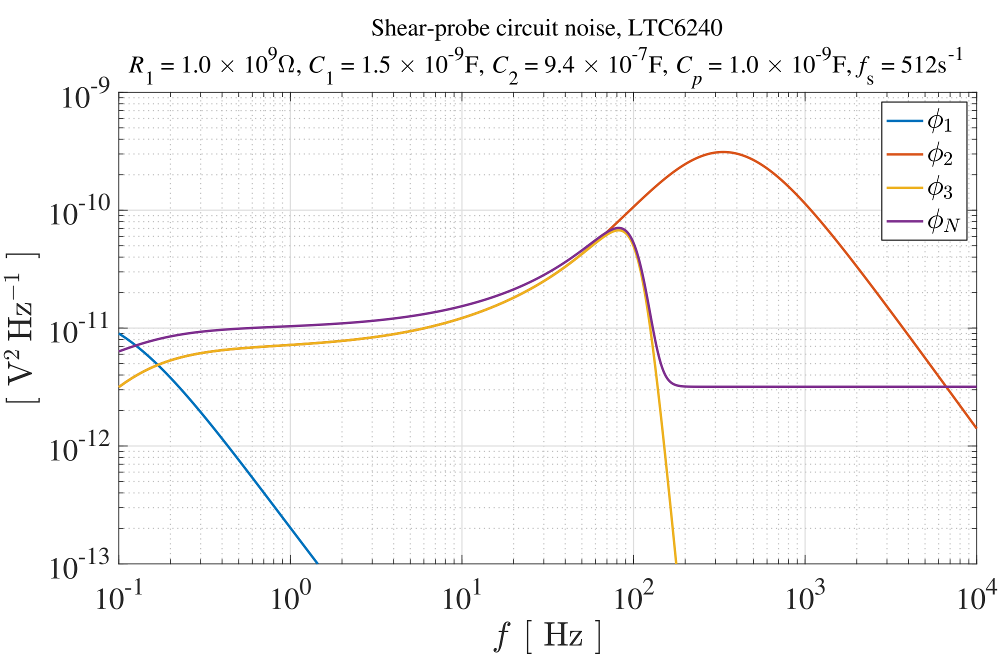

Figure 8: Same as Figure 7 but with shear-probe connected.

3 Comparison of noise model and measured data

Several comparisons between the noise model and measured spectra from the shear channel were made (Figure 9). The parameters for the input voltage noise model (E_1 and f_c) were modified slightly from the spec sheet values to better agree with the more than 200 noise spectra generated during ASTP board calibrations. The modified values are:

Item	Value	Description
E_1	$9 \times 10^{-9}\,\mathrm{V}/\sqrt{\mathrm{Hz}}$	LTC-6240 input voltage noise at high frequency
f_c	50 Hz	LTC-6240 flicker noise knee frequency

Table 2: Modified parameters for the LTC-6240 to obtain better agreement with measured signals.

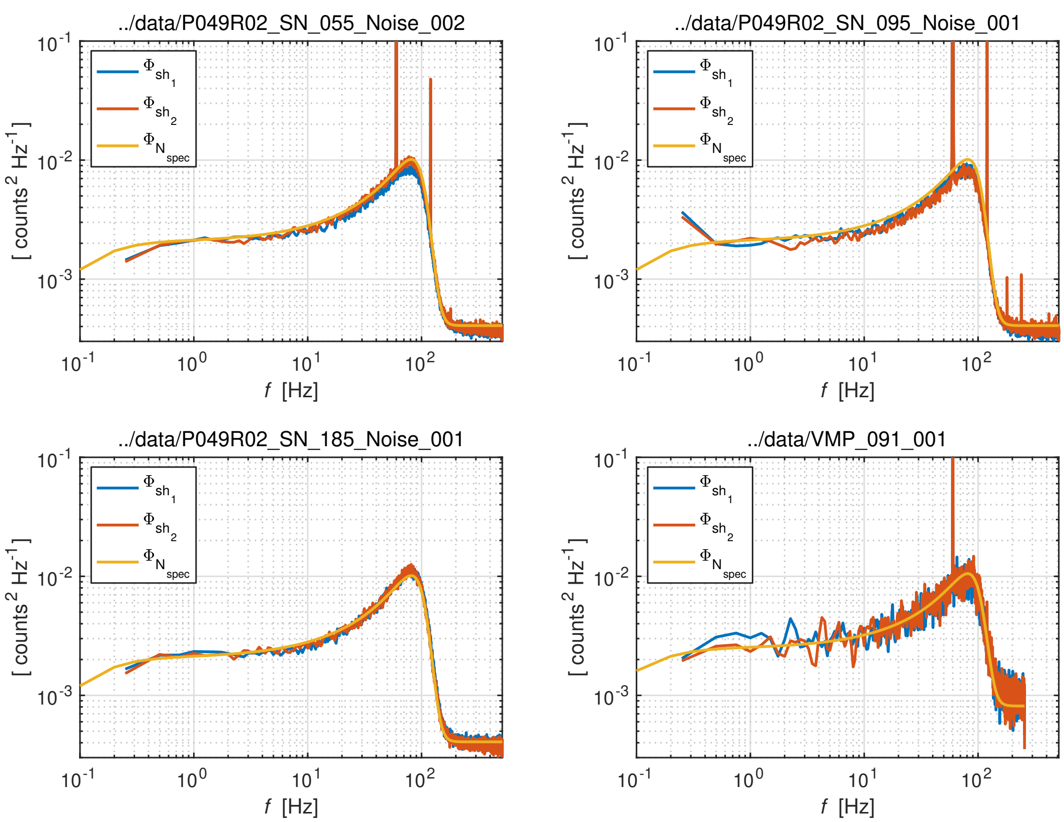

Figure 9: Comparisons between the modelled and measured noise spectra, when no probe is connected to the circuitry ($C_p = 0\,\mathrm{nF}$). Three of the data files (top left, top right, bottom left) were collected during ASTP board calibrations when the sampling rate was 1024 Hz. One data file (bottom right) was collected during a 60-s bench using a sampling rate of 512 Hz.

4 Checking the shear-channel noise in your instrument

The main reason for collecting shear-probe noise data, without a probe attached ($C_p = 0\,\mathrm{nF}$), is to check the electronics of your instrument. All instruments are checked for noise at RSI before shipping by collecting data without a probe connected. The spectra of the noise are displayed in the calibration report for your instrument (usually in the very last figure of this report). If you want to check the noise of the electronics, then you do this by disconnecting the shear-probes and connecting the "test" probes that are supposed to be installed whenever your instrument is not used for measurements (shipping, storage, etc.). The appropriate test probes are marked `S1` and `S2`. They leave the shear-probe channel inputs electrically disconnected but provide shielding to the internal cabling of your instrument.

To check your instrument, collect one to two minutes of data with the instrument horizontal, preferably cushioned by soft foam to minimize vibrations, and with the ON/OFF magnet (or the pressure port) near top dead centre. The Matlab function `quick_bench` (in your data processing library) will provide a spectrum of signal noise. This spectrum should not differ by more than a factor of $\sim$3 from the one shown in the calibration report for your instrument, or in the examples (and model) shown in Figures 9 and 5.

5 Conversion to physical units

If you want to estimate the electronics noise in your shear-probe data, then you want to start with the noise model that includes the effects of the probe capacitance, $C_p = 1\,\mathrm{nF}$ (Figure 6). This spectrum is not in physical units of shear. Rather it is in units of $\mathrm{V}^2\,\mathrm{Hz}^{-1}$ for ϕ_N (14) , and in units of counts^2/Hz for ψ_N (15).

The conversion of ϕ_N into physical units is given by

$$\begin{aligned} \phi_{\nabla v}(f) &= \beta^{-2}\,\phi_N(f) \\ \beta &= 2\sqrt{2}SG_DU^2 \end{aligned} \tag{16}$$

where $G_D = (R_3C_2)^{-1} \approx 1\,\mathrm{s}^{-1}$ is the differentiator gain which is provided in the calibration report of your instrument, $S{\sim}0.07\,\mathrm{V/(m/s)^2}$ is the sensitivity of the shear probe which is provided in the calibration report for the probe, and U is the speed of profiling in units of $\mathrm{m\,s^{-1}}$. The spectrum $\phi_{\nabla v}(f)$ is the *frequency* spectrum of shear noise. The wavenumber spectrum of noise is $\phi_{\nabla v}(k) = U\phi_{\nabla v}(f)$, where the wavenumber is $k = f/U$.

Clearly, the spectrum of electronic noise, expressed in units of shear, is strongly dependent on the speed of profiling. The frequency spectrum scales with U^{-4} and the wavenumber spectrum scales with U^{-3}. Slow speeds give a high noise spectrum. The noise variance, which is the integral of the noise spectrum, always scales with U^{-4}.

6 Using the noise_shearchannel function

The Matlab function `noise_shearchannel` can be used to generate a noise spectrum using the model of (15). The syntax for calling this function is:

```
[result, params] = noise_shearchannel(f, varargin)
```

where `f` is a frequency vector and `varargin` is either a structure or a collection of string-value pairs that can be used to override the default values. Using `varargin` is optional.

The output of the function, `result`, is the noise frequency spectrum of the shear signal in units of counts^2/Hz. The optional output `params` are the model parameters used to compute the spectrum.

The default parameters for the model can be viewed by calling the function without any input arguments. The default values are returned in a structure that you can modify and save for later usage. For example,

```
>> noise_shearchannel
ans =
  struct with fields:
            Bits: 16
              C1: 1.5000e-09
              C2: 9.4000e-07
              C3: 4.7000e-10
              CP: 0
             E_1: 9.0000e-09
             I_1: 5.6000e-16
             K_B: 1.3820e-23
              R1: 1.0000e+09
              R2: 499
              R3: 1000000
             T_K: 295
             VFS: 4.0960
            f_AA: 110
              fc: 50
              fs: 512
       gamma_RSI: 2.5000
    make_figures: 0
>>
```

The optimization discussed in section 3 is incorporated into the default values.

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